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WINGBOX • Code Details V1

WINGBOX

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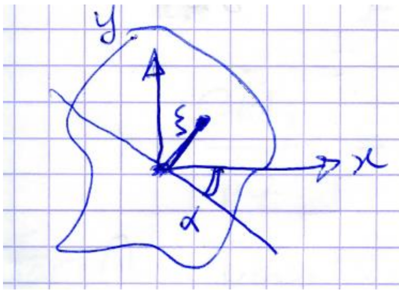
Contents

Basic Theory of Stressed Skin Design

WINGBOX deals with the estimation of the equilibrium stresses and associated displacements (small strains) in multicell semi-monocoque aeronautical structures in accordance to Engineering Bending Theory and Bredt-Batho Theory.

Basic Engineering Bending Theory

For a prismatic beam section, the following figure defines the Bending Moments as a function of the axial stresses and their distance from the neutral axis:



$$\xi = x \sin\alpha + y \cos\alpha$$

$$M_x = \int_A \sigma_z \, y \, dA \qquad M_y = \int_A \sigma_z \, x \, dA$$

while the sectional properties are as below:

$$I_{xx} = \int_A y^2 dA \qquad I_{yy} = \int_A x^2 dA \qquad I_{xy} = \int_A xy \, dA$$

The sectional axial stresses are then as below as a function of the effective bending moments:

$$\sigma_z = \frac{\bar{M}_x}{I_{xx}} y + \frac{\bar{M}_y}{I_{yy}} x$$

and the effective bending moments as below:

$$\bar{M}_x = \frac{M_x - M_y I_{xy}/I_{yy}}{1 - I_{xy}^2/I_{xx}I_{yy}} \qquad \bar{M}_y = \frac{M_y - M_x I_{xy}/I_{xx}}{1 - I_{xy}^2/I_{xx}I_{yy}}$$

If the bending moments are produced by Shear Forces, then the following equivalent shear forces will be defined:

$$\bar{S}_x = \frac{S_x - S_y I_{xy}/I_{xx}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} \qquad \bar{S}_y = \frac{S_y - S_x \frac{I_{xy}}{I_{yy}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

And if the shear forces are the result of Distributed Loads along the z-axis (axial) then the following equivalent distributed loads will be defined:

$$\bar{W}_x = \frac{W_x - W_y \frac{I_{xy}}{I_{xx}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}} \qquad \bar{W}_y = \frac{W_y - W_x \frac{I_{xy}}{I_{yy}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

Bredt-Batho Theory for an Open Tube

We can define the shear flow q as the product of the shear stress with the thickness of the shell. It can then be shown that the shear flow gradient around the section depends on the axial stress gradient which in turn depends on the applied shear loads.

$$\frac{\partial q}{\partial s} + t\,\frac{\partial \sigma_z}{\partial z} = 0$$

This leads to the following relationship for the shear flow and the applied equivalent shear forces when they are applied through the Shear Center of the section:

$$q_s = -\frac{\bar{S}_y}{I_{xx}}\int_0^s t\,y\,ds - \frac{\bar{S}_x}{I_{yy}}\int_0^s t\,x\,ds$$

The location of the Shear Centre can be found by equation the moments of the shear forces about a convenient point to the moments produced by the above shear flow distribution:

$$S_y\,\xi_{E_s} = \text{Moment}$$

$$S_x\,\eta_{E_s} = \text{Moment}$$

Bredt-Batho Theory for a Closed Tube

A closed tube is treated in a similar way as the open tube, since the basic differential equation relationship between the shear flow gradient and the axial stress gradient is the same. The only difference now is that since the tube is closed there is no preferred initial value for the integration of the shear flow. In the case of the open tube, the integration starts from an open end with a ZERO initial value.

The process then is to virtually “cut” the tube and assume an extra unknown value of an initial shear flow (which is then just a constant over the section).

$$q_s = q_{open} + q_{s,0}$$

By “cutting” the tube we can readily use the formula of the previous section for the shear flow distribution. What remains is the unknown initial value, which is found by moment equilibrium considerations with the applied shear loading:

$$S_x \eta_0 - S_y \xi_0 = \oint p\, q_{open}\, ds + 2A q_{s,0}$$

where A is the cross sectional area of the tube:

$$2A = \oint p\, ds$$

The Shear Center of the closed tube can now be found in a similar way as the open tube by moment equilibrium about a convenient point,

$$S_x \eta_0 = \oint \left(q_{open} + q_{s,0} \right) p\, ds$$

$$S_y \xi_0 = \oint \text{-------- ---}$$

the difference being that the value of the constant shear flow will now be obtained from “zero section rotation” considerations, since we are dealing with the shear center:

$$\frac{d\theta}{dz} = 0 = \oint \frac{q_s}{Gt}\, ds$$

$$q_{s,0} = -\frac{\oint q_{open}/Gt \, ds}{\oint \frac{ds}{Gt}}$$

Torsion of Closed Tubes

It can be shown that for a closed tube under torsion, the shear flow is constant over the section:

$$T = 2Aq$$
$$2A = \oint p \, ds$$

and the resulting rotation gradient:

$$\frac{d\theta}{dz} = \frac{T}{4A^2} \int \frac{ds}{Gt}$$

Effect of Booms

Booms are idealized entities that represent stiffeners and flanges as concentrated areas B that carry only direct stresses, in our case, axial stresses. The shear stress of the skin is then affected by a change caused by the "axial stress discontinuities" from the presence of the booms.

For an Open Tube then the total shear flow distribution due to r number of booms will be:

$$q_s = -\frac{\bar{S}_y}{I_{xx}}\left(\int_0^s t_D y \, ds + \sum_r B_r y_r\right) - \frac{\bar{S}_x}{I_{yy}}\left(\int_0^s t_D x \, ds + \sum_r B_r x_r\right)$$

where the thickness used is that of the axial load carrying panels.

Similarly for a Closed Tube with r booms:

$$q_s = -\frac{\bar{S}_y}{I_{xx}}\left(\int_0^s t_D y\,ds + \sum_r B_r y_r\right) - \frac{\bar{S}_x}{I_{yy}}\left(\int_0^s t_D x\,ds + \sum_r B_r x_r\right) + q_{s,0}$$

And the unknown shear flow $q_{s,0}$ is found like before.

Deflection and Rotation of Open and Closed tubes

The deflections and rotations at particular points along the axis of an open or a closed tube can be found using the Unit Load Method.

This is done by equating the external work done by a Virtual Unit Load applied in the direction of the deflection or rotation due to the real loads that we want to find and equate it with the corresponding internal work done by the associated Virtual "Unit Load" Stresses over the real structural strains.

Therefor for deflections due to bending moment distribution M:

$$\underbrace{1 \cdot \Delta_M}_{\text{unit load in direction of } \Delta_M} = \int_L \left(\int_A \sigma_{z,1}\, \varepsilon_{z,0}\, dA \right) dz$$

For deflections due the applied shear forces:

$$\Delta_s = \int_L \left(\int_s \tau_1 \gamma_0 t\, ds \right) dz$$

For deflections due to torques:

$$\Delta_T = \int_L \frac{T_0 T_1}{GJ}\, dz$$

Rotations can be found in similar fashion through virtual unit moments.

Multi Cell Tubes

All the above is valid for Single Cell tubes. For Multi Cell Tubes the previous concepts apply but for each cell individually. The Multi Cell is then obtained by enforcing compatibility conditions.

Multi Cells under Torsion

Assuming N cells, the total torque applied links to the individual cell shear flows as below:

$$T = \sum_{R=1}^{N} 2A_R q_R$$

While for each R cell we can express the rotation gradient in terms of the interface shear flows and the Rth shear flow, as below:

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}\left[-q_{R-1}\,\delta_{R-1,R} + q_R\,\delta_R - q_{R+1}\,\delta_{R+1,R}\right]$$

With

$$\delta_{R-1,R} = \int_{\text{common wall}} \frac{ds}{t} \qquad \delta_R = \oint_R \frac{ds}{t}$$

We can then write N equations as above with N+1 unknowns (the N shear flows and the rotation gradient). Combining with the Torque equation, all the unknowns can be found.

Multi Cells under Shear Loads

In a similar way with the Single Cell closed tubes, we assume each cell be fictitiously “cut” and the resulting equilibrium shear flows will be, for each cell of the Mutli Cell tube:

$$q_{open} = -\frac{\bar{S}_y}{I_{xx}}\left(\int_0^s t_D\, y\, ds + \sum B_r y_r\right) - \frac{\bar{S}_x}{I_{yy}}\left(\int_0^s t_D\, x\, ds + \sum B_r x_r\right)$$

The above shear flows are obtained as if the shear loads were applied at the shear center of the section. Hence there is a need to add an extra shear flow for each tube due to a torque contribution and this will be the shear flow missing at the “cut”, just like before.

Assuming N cells, the rotation gradient of the section will be for each R cell a function of the Rth shear flow and the interface shear flows as below:

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}\left(-q_{s0,R-1}\,\delta_{R-1,R} + q_{s0,R}\,\delta_R - q_{s0,R+1}\,\delta_{R+1,R} + \oint_R q_{open}\frac{ds}{t}\right)$$

With

$$\delta_{R-1,R} = \int_{\text{common wall}} \frac{ds}{t} \qquad \delta_R = \oint_R \frac{ds}{t}$$

We can then write N equations as above with N+1 unknowns (the N shear flows and the rotation gradient). Combining with the Moment equilibrium equation of the applied shear loads about a convenient point (as below), all the unknowns can be found.

$$S_x \eta_0 - S_y \xi_0 = \sum_{R=1}^{N} \oint_R q_{open,R}\, p_0\, ds + \sum_{R=1}^{N} 2A_R\, q_{s0,R}$$

Multi Cells with Taper

Taper is considered to be the effect of the projection of the axial zz stresses to the plane of the section hence modifying the effective shear loads. It is best considered through the effect of booms as they are by large the biggest axial stress carriers.

The axial stresses in a Multi Cell tube are as (for any type of prismatic tube):

$$\sigma_{z,r} = \frac{\overline{M}_x}{I_{xx}} y_r + \frac{\overline{M}_y}{I_{yy}} x_r$$

whatever the origin those bending moments in the section.

Then the applied shear loads will be modified by the projected boom loads as below:

$$\underbrace{S_{x,w}}_{\text{web}} = S_x - \sum P_{z,r} \frac{\delta x_r}{\delta z} \qquad S_{y,w} = S_y - \sum P_{z,r} \frac{\delta y_r}{\delta z}$$

where:

$$P_{z,r} = \sigma_{z,r} B_r$$

The corresponding open section cell shear flows (fictitious “cuts”) will then be:

$$q_{open} = -\frac{\bar{S}_{y,w}}{I_{xx}}\left(\int_0^s t_D\, y\, ds + \sum B_r y_r\right) - \frac{\bar{S}_{x,w}}{I_{yy}}\left(\int_0^s t_D\, x\, ds + \sum B_r x_r\right)$$

The sectional rotation gradient equations do not change.

However, the shear loads moment equilibrium equation is updated as below:

$$S_x \eta_0 - S_y \xi_0 = \sum_R^N \oint p_0\, q_{open,R}\, ds + \sum_R^N 2A_R\, q_{s0,R} - \sum P_{y,r} \xi_r - \sum P_{x,r} \eta_r$$

with

$$P_{x,r} = P_{z,r} \frac{\delta x_r}{\delta z}$$

$$P_{y,r} = P_{z,r} \frac{\delta y_r}{dz}$$