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PROFILE™ • Code Details V1

PROFILE

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Introduction

PROFILE is an analytical code that deals with the prediction of penetration and/or perforation of targets under a Kinetic Energy Impactor, usually in the hypervelocity range.

In fact, it deals with impact scenarios where the impactor is totally consumed by its collision with the target. In such a case the impactor and the impacted area in the neighbourhood of the target, appear to behave like “fluids”.

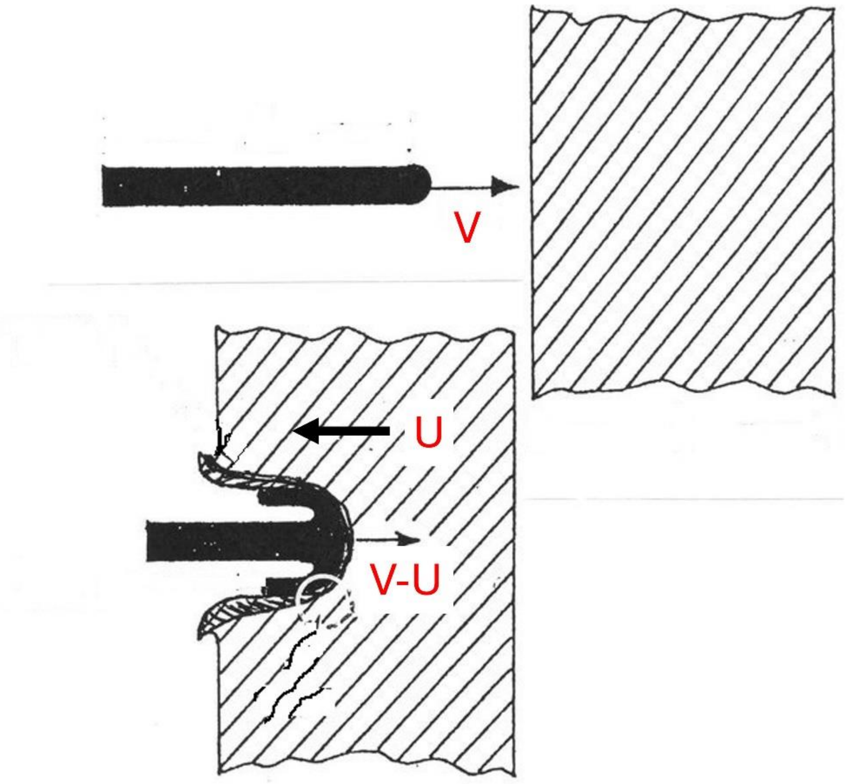
The extreme strain rates during these phenomena imply that the material strength plays a lesser role as compared to that of the momentum deposited (material densities).

Penetration and Cratering

The basic assumption in this part of the code is that the impactor is finite and axisymmetric while the target is semi-infinite. It is also assumed that the relative speed between the impactor and the target is greater than 5 km/s, that is, a speed high enough for both materials to behave as fluids since the collision pressures will be well above any dynamic material strength.

We define an impactor hitting a target with speed V, resulting in a crater been created at a speed U.

The phenomenon can be defined with respect to the bottom of the crater as it is formed. Assuming that the crater bottom is stationary, we can observe the target approaching with a speed U while the impactor approaches with a speed V-U with respect to the crater bottom.



From the Bernoulli hydrodynamic theory, the stagnation pressure at the impactor / target interface can be defined as Ps

$$P_s = \frac{1}{2}\rho_i (V-U)^2 + \sigma_i = \frac{1}{2}\rho_t U^2 + \sigma_t$$

where

P_s = stagnation pressure at the interface

ρ_i = impactor material density

V = impactor velocity at interface

U = target velocity at interface

σ_i = dynamic strength of the impactor

ρ_t = target material density

σ_t = dynamic strength of the target

Two basic assumptions are necessary to apply the Bernoulli theory:

- the impactor and the target behave as incompressible fluids during the penetration process.
- flow is assumed to be steady state when viewed from a frame of reference moving with the penetration velocity, U, so that for the streamline along the axis of symmetry, the stagnation pressure at the impactor / target interface is Ps.

Also the phenomenon is assumed to be axisymmetric with only axial speed evolution and no radial speed explicitly used, although its effect is taken care of implicitly by a volume – versus-kinetic energy deposition relationship.

Rearranging and then solving for U, the target velocity at the interface, yields the penetration rate, dP/dt. This is the velocity of the bottom of the hole in the target.

$$\frac{dP}{dt} = \frac{-2\rho_i V \pm \sqrt{4\rho_i^2 V^2 - 4(\rho_t - \rho_i)(\sigma_t - \sigma_i - \rho_i V^2)}}{2(\rho_t - \rho_i)}$$

The impactor is artificially segmented into a user-defined number of axial elements. The above equations are then applied in each element sequentially from the tip of the impactor to its rear.

The kinetic energy, momentum, mass, tip velocity, and tail velocity for this impactor element is calculated. The penetration time of the impactor element is then obtained from its size and differential velocities from tip to tail. Once found, the above equation can be integrated to give the penetration increment. At the end of the penetration process for this element, the total depth of penetration in the target will have increased by DP.

However, this DP will not take into account interaction with the crushed walls of the target that might decrease its real value, or phase changes from energy deposition effects that might increase its real value.

Hence the actual penetration increment DP will be factored by a scale factor that needs to be defined from experimental data or previous experience. It is this scaled DP that will be used to increment the total penetration P.

Once the penetration increment DP was found, the hole size attributed to this element is found by assuming a cylindrical hole segment of depth DP and radius R.

Hence the hole radius at that depth increment will be found from the volume increment DV

$$D\forall = \pi R^{2} DP$$

The volume increment can be obtained either from the Kinetic Energy deposited by the impactor element or from the crushing strength of the target, whichever is the most significant.

In the case of kinetic energy, a linear dependence of the volume increment to the element kinetic energy has been suggested and used by many researchers:

$$E^{\text{element}}_{\text{Kinetic}} \approx \text{Factor} \times D\forall$$

Alternatively, the stagnation pressure at the impactor – target interface is related to the crushing pressure over a hemisphere of radius R.

The crater volume will be further enhanced, if needed, by the spallation effect, if present. In this respect, the elastic waves propagating through the target towards the surface will interfere and if the spall strength is reached, the entrance of the crater will increase accordingly.

This is true if the impact is upon soil or concrete and especially if at an angle less than 90 degrees.

Impactors that have a uniform axial velocity will correspond to “long rods”. However, “pre-segmented long rods” are not dealt for the time being, maybe at a later stage.

Impactors that have a non-uniform but sequentially decreasing axial velocity from tip to rear, they will correspond to the “collapsed liner” types. In such a case, a “cut-off” velocity will be defined to limit the application of the Bernoulli theory and ignore it for elements that correspond to the “slug”. A “particulation time” might need to be defined if the effect of the axial strain rate is excessive with respect to the target distance.

Perforation in the Hypervelocity Range

This part deals with the case of Space Debris Impact upon thin targets, normally the shielding shell around a spacecraft.

The Shock-Hugoniot (or Rankine-Hugoniot) state of the phenomenon is estimated.

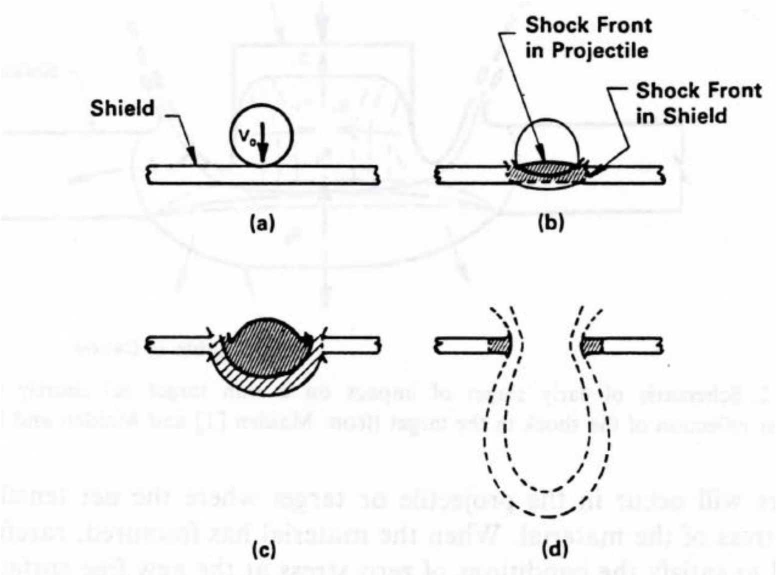


FIG. 3. Schematic of formation of debris bubble (after Riney and Halda

The first step is to establish the particle speed Up which can be found from the momentum balance from the bulk compression of the interface between the impactor and the target. It depends largely to the material densities of the impactor and the target.

Once estimated then the bulk compression shock speed will be estimated using the following formula that has been experimentally proposed and used by all researchers since many decades.

$$U_s = c + sU_p$$

where C is the bulk compression speed of sound of the material in question and S is an experimentally found factor.

It can then be shown that the Shock-Hugoniot pressure will be as below:

$$P_H = \frac{\rho_0 c^2 x}{(1-sx)^2}$$

where the compressibility factor X will be as below:

$$x = 1 - V/V_0$$

The corresponding internal energy will be:

$$\varepsilon_H = \frac{P_H}{2}(V_0 - V) = \frac{P_H V_0 x}{2}$$

from which the corresponding temperature of the “shocked” material will be:

$$T_H = (\varepsilon_H - \varepsilon_0) / 3R$$

where **R** is the universal gas constant per mole of the material and ε_0 is the zero Kelvin internal energy as below:

$$\varepsilon_0 = \varepsilon_{00} + \varepsilon_{01} x + \varepsilon_{02} x^2 + \varepsilon_{03} x^3 + \varepsilon_{04} x^4$$

The above coefficients can be estimated theoretically from the shock condition coefficients C and S, from R and from the Grüneisen parameter Γ_0 at zero Kelvin state.

After the shock has passed, the residual temperature T_{exp} can be estimated as below from the expansion isentrope:

$$T_H = T_{Exp}\, e^{\rho_0 \Gamma_0 (V_0 - V_H)}$$

A range of equations are then proposed to estimate the diameter of the perforated hole as a function of the impactor (relative to the target) velocity, the impactor equivalent diameter, the target thickness, the material densities etc.

Nysmith formula

General Motors formula

Sawle formula

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